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COATING OF A NON-NEWTONIAN FLUID ONTO A MOVING SURFACE

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The application of a coating of non-Newtonian fluids to a moving surface was considered in [1-4]. These studies are based on the approach proposed in [5, 6], which is restricted by the requirement that the thickness h_0 of the coated film be small as compared with the capillary constant $(\sigma/\rho g)^{1/2}$ (ρ is the density, g the free-fall acceleration, and σ the surface tension). Experimental studies of fluid coating [1, 7] have revealed substantial differences between the theoretical and experimental data. Thus at present we lack a theory that satisfactorily describes the coating of non-Newtonian fluids. In this article such a theory is developed for fluids with nonlinear viscosity.

1. Let us consider the process of coating a liquid onto a vertical surface moving at a constant speed (Fig. 1). Because of the action of gravity, the withdrawn plate entrains only part of the liquid it sets in motion. Accordingly, on the free surface there is a stagnation line (perpendicular to the plane of the drawing) where the velocity is equal to zero and the flow direction branches [8]. The streamlines passing through the stagnation line separate the wall zone of liquid entrained by the wall from that remaining in the bath.

We take the stagnation line as the origin and direct the x axis vertically upward in the direction of motion of the surface, and the y axis at right angles to the latter. The flow region bounded below by a plane perpendicular to the wall and passing through the stagnation line and tending upward to the constant thickness h_0 we will call the dynamic meniscus zone. Clearly, the length L of the dynamic meniscus zone considerably exceeds its width h_0; this naturally gives rise to the small parameter $\varepsilon = h_0/L \ll 1$. Consequently, the variation of the characteristics along the x axis is much weaker than in the transverse y direction, i.e., the derivatives with respect to y are much greater than those with respect to x. Making the appropriate estimates [9] in the equations of motion and the boundary conditions, in the region of the dynamic meniscus we obtain

$$\partial\tau/\partial y + \rho g - \partial p/\partial x = 0,\ \partial p/\partial y = 0; \tag{1.1}$$

$$u = U \text{ when } y = 0,\ \tau = 0 \text{ when } y = h; \tag{1.2}$$

$$p - p_0 = -\sigma d^2h/dx^2 \text{ when } y = h. \tag{1.3}$$

We represent the continuity equation in integral form: $Q = \int_0^h u\,dy =$ const. Here τ is the shear stress due to friction; h is the coordinate of the free surface of the liquid; p is the pressure in the liquid; u is the x component of the velocity vector; p_0 = const is the pressure in the gas; and Q is the rate of flow of the liquid in the film.

Integrating (1.1) with respect to y and using (1.2) and (1.3), we find

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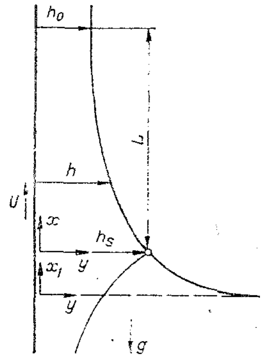

Fig. 1

$$\tau = -(\rho g - \sigma d^3h/dx^3)(h - y). \tag{1.4}$$

As the rheological equation of state we will take the widely used power law

$$\tau = k\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}, \tag{1.5}$$

where k is the consistency coefficient; and n is the parameter characterizing non-Newtonian behavior. As a result of the substitution of (1.4) in (1.5) and integration using the first of equations (1.1), we obtain

$$u = U - \frac{n}{n+1}\left(\frac{\rho g}{k} - \frac{\sigma}{k}\frac{d^3h}{dx^3}\right)^{\frac{1}{n}}\left[h^{\frac{n+1}{n}} - (h-y)^{\frac{n+1}{n}}\right]. \tag{1.6}$$

Here the fact that in the dynamic meniscus zone $\partial u/\partial y < 0$ has been taken into account. We write the flow rate

$$Q = \int_0^h u\,dy = Uh - \frac{n}{2n+1}h^{\frac{2n+1}{n}}\left(\frac{\rho g}{k} - \frac{\sigma}{k}\frac{d^3h}{dx^3}\right)^{1/n}. \tag{1.7}$$

In the region of constant film thickness h_0 all the derivatives with respect to the x coordinate are equal to zero; then

$$Q = Uh_0 - \frac{n}{2n+1}\left(\frac{\rho g}{k}h_0^{2n+1}\right)^{1/n}. \tag{1.8}$$

Substitution of (1.8) in (1.7), and going over to the dimensionless variables and parameters

$$H = \frac{h}{h_0},\ z = \frac{x}{h_0},\ T = h_0\left(\frac{\rho g}{kU^n}\right)^{\frac{1}{1+n}},\ \mathrm{Ca} = \frac{k}{\sigma}U^n h_0^{1-n} \tag{1.9}$$

leads to an ordinary nonlinear differential equation for the film thickness H in the dynamic meniscus zone:

$$\frac{H^{2n+1}}{\mathrm{Ca}}\frac{d^3H}{dz^3} = T^{n+1}H^{2n+1} - \left[\frac{2n+1}{n}(H-1) + T^{\frac{n+1}{n}}\right]^n. \tag{1.10}$$

This equation determines the shape of the film surface in the region of variation of h from h_0 to h_S, where h_S is the thickness of the liquid layer at the stagnation line. The integration of (1.10) produces three arbitrary constants. These can be found from the condition governing the behavior of the solution as $z \to \infty$:

$$H \to 1,\ dH/dz \to 0,\ d^2H/dz^2 \to 0. \tag{1.11}$$

Equation (1.10) contains another unknown, namely h_0, which is linked with the flow rate by relation (1.8). In order to calculate this quantity, we determine the shape of the liquid surface below the stagnation line and then join it to the shape of the dynamic meniscus. The joining condition is the missing condition needed to calculate h_0.

First we find the position of the stagnation line. By definition, on that line the velocity $u|_{y=h}$ of the film surface is equal to zero; then, starting from (1.6),

$$0 = U - \frac{n}{n+1} h_s^{\frac{n+1}{n}} \left(\frac{\rho g}{k} - \frac{\sigma}{k}\frac{d^3 h_s}{dx^3}\right)^{1/n}. \tag{1.12}$$

On the other hand, in view of the constancy of the flow rate Q in each cross section of the film from (1.7) and (1.8) there follows

$$Uh_0 - \frac{n}{2n+1} h_0^{\frac{2n+1}{n}} \left(\frac{\rho g}{k}\right)^{1/n} = Uh_s - \frac{n}{2n+1} h_s^{\frac{2n+1}{n}} \left(\frac{\rho g}{k} - \frac{\sigma}{k}\frac{d^3 h_s}{dx^3}\right)^{1/n}. \tag{1.13}$$

Thus, we have a system of two equations (1.12) and (1.13) with two unknowns h_s and $\rho g - \sigma d^3 h_s/dx^3$. Solving for h_s and going over to the dimensionless variables (1.9), we obtain the position of the stagnation line

$$H_s = (2n+1)/n - T^{(n+1)/n}, \tag{1.14}$$

where $H_s = h_s/h_0$.

2. Let us consider the region of flow of the liquid located below the stagnation line, which remains in the bath. Here the space derivatives of the velocities and stresses are much smaller than in the dynamic meniscus zone, and in the equations of motion and boundary conditions they can be neglected as compared with the forces of gravity and surface tension. Then the shape of the surface is given by the relations

$$\partial p/\partial x_1 + \rho g = 0, \; \partial p/\partial y = 0 \tag{2.1}$$

with boundary conditions

$$p - p_0 = -\sigma \frac{d^2 h}{dx_1^2}\left[1 + \left(\frac{dh}{dx_1}\right)^2\right]^{-3/2} \text{ when } y = h. \tag{2.2}$$

Here the axis x_1 coincides with the x axis, but is reckoned from the horizontal surface of the liquid in the bath (see Fig. 1). From (2.1) and (2.2) we obtain the equation

$$\frac{d^2 h}{dx_1^2}\left[1 + \left(\frac{dh}{dx_1}\right)^2\right]^{-3/2} = \frac{\rho g}{\sigma} x_1 \tag{2.3}$$

of the equilibrium shape of the liquid surface in the gravity field. We will call the zone below the stagnation line the static meniscus region.

For a stationary wall the equation (2.3) should be integrated with the conditions

$$dh/dx_1 \to -\infty \text{ as } x_1 \to 0 \tag{2.4}$$

(the surface of the liquid remote from the plate is horizontal), and

$$h = 0 \text{ when } x_1 = x_0. \tag{2.5}$$

Here, in the case of a completely wetted wall x_0 the height to which the liquid rises up the wall under the action of capillary forces is found from

$$dh/dx_1 = 0 \text{ when } x_1 = x_0. \tag{2.6}$$

In the case of a moving wall it is necessary to take into account the effect of the entrained film on the shape of the surface in the static meniscus zone. We assume that the static meniscus is in contact not with a solid wall but with a liquid film of thickness h_0. Then conditions (2.4) and (2.6) remain unchanged, while (2.5) takes the form

$$h = h_0 \quad \text{when } x_1 = x_0. \tag{2.7}$$

Integrating (2.3) with allowance for (2.4), (2.6), and (2.7) and going over to the dimensionless quantities (1.9), we find

$$(\mathrm{Ca}\, T^{n+1})^{1/2}\,(H-1) = \sqrt{2} - \frac{1}{2}\ln\frac{\sqrt{2}+1}{\sqrt{2}-1} - (4-\mathrm{Ca}\,T^{n+1}z_1^2)^{1/2} + \frac{1}{2}\ln\frac{2+(4-\mathrm{Ca}\,T^{n+1}z_1^2)^{1/2}}{2-(4-\mathrm{Ca}\,T^{n+1}z_1^2)^{1/2}}, \qquad (2.6)$$

where $z_1 = x_1/h_0$.

3. In order to determine the thickness T of the coated film, we join solution (2.6) for the static meniscus zone and the solution of the dynamic meniscus equation (1.10). Obviously, on the stagnation line the stress in the liquid must be the same whether determined from the dynamic or the static zone. Then, starting from (1.3) and (2.2) and equating the right sides, we obtain the joining condition

$$\left.\frac{d^2h}{dx^2}\right|_{h_s} = \frac{d^2h}{dx_1^2}\left[1+\left(\frac{dh}{dx_1}\right)^2\right]^{-3/2}\Bigg|_{h_s}, \qquad (3.1)$$

which, using (2.3), we write in the dimensionless form

$$\left.\frac{d^2H}{dz^2}\right|_{H_s} = \mathrm{Ca}\,T^{n+1}z_1\big|_{H_s}. \qquad (3.2)$$

Thus, the procedure for solving the problem is as follows. We assign the parameter Ca and select a certain starting value of T; from (1.14) we obtain H_s, and then numerically (for example by the Runge–Kutta method) integrate Eq. (1.10) with conditions (1.11). For integration purposes it is necessary to know the asymptotic behavior of the solution as $z \to \infty$. For sufficiently large z we can put

$$H(z) = 1 + \gamma(z), \qquad (3.3)$$

where $\gamma(z)$ is a small quantity. Substituting (3.3) in Eq. (1.10) and retaining only small terms of the first order in γ, we find

$$d^3\gamma/dz^3 = -(2n+1)\mathrm{Ca}\,T^{(n^2-1)/n}(1 - T^{(n+1)/n})\,\gamma.$$

Hence $\gamma = A\exp(-Bz)$ and, consequently,

$$dH/dz = -B\gamma,\ d^2H/dz^2 = B^2\gamma, \qquad (3.4)$$

where $B = [(2n+1)\mathrm{Ca}T^{(n^2-1)/n}(1 - T^{(n+1)/n})]^{1/3}$. Conditions (3.3) and (3.4) are the initial data for the integration of (1.10). By calculating the second derivative d^2H/dz^2 for $H = H_s$ and then determining from (2.8) the value of the coordinate z_1 at which $H = H_s$, we check the satisfaction of joining condition (3.2). By a simple iteration method we find the value of T satisfying this condition for the given Ca.

The results of the calculations are reproduced in Fig. 2, where they are compared with the most accurate experimental data obtained in [7, 10]. The theoretical curves I-IV correspond to n = 1, 0.8, 0.6, and 0.4. The points 1 (n = 1) are the data of [9], and the points 2-7, which correspond to n = 0.61, 0.545, 0.575, 0.367, 0.374, and 0.393, are the results of [7].

For a Newtonian liquid (n = 1) there is good agreement between experiment and theory over the entire interval of withdrawal rates Ca. We note that when Ca > 3 we have T = 0.78 or $h_0 = 0.78(\mu U/\rho g)^{1/2}$ (μ is the dynamic viscosity), i.e., the film thickness ceases to depend on the surface tension. If Ca < 3, then the surface tension has a substantial influence on the thickness of the coated layer T.

Figure 2 shows that curves I-IV for Newtonian and pseudoplastic fluids lie very close together in the region of small and intermediate (Ca < 1) withdrawal rates and separate significantly only at large (Ca > 1) withdrawal rates, when it is possible to neglect the effect of surface tension. The experimental points 2-7 for pseudoplastic fluids are not consistent with the curves III and IV predicted by the theory. We will examine the reasons for this discrepancy.

4. As is known, power law (1.5) does not describe the rheological behavior of a fluid at low shear rates. In the opinion of the authors of [2], this feature of the power-law model may prove important in withdrawal problems, where in the dynamic meniscus zone near the free surface of the fluid the shear rate is very low. We will make use of the Ellis model

$$\partial u/\partial y = \tau(a + b|\tau|^{\alpha-1}) \qquad (4.1)$$

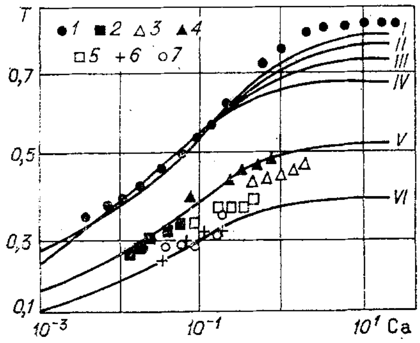

Fig. 2

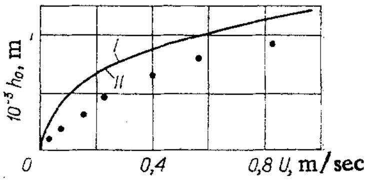

Fig. 3

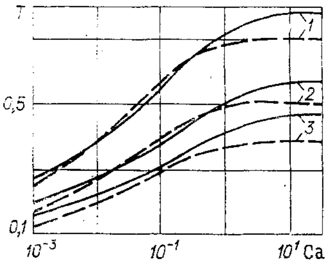

Fig. 4

(a, b, and α are rheological parameters), which adequately describes the viscous properties of non-Newtonian fluids over a broad range of shear rates (much greater than in the case of the power law) and can be employed in the region of low shear stresses. Basing ourselves on the above approach to withdrawal problems and using model (4.1), we obtain:

for the dynamic meniscus

$$\frac{H^3}{3\,\mathrm{Ca}}\frac{d^3H}{dz^3}-\frac{T_1^{\alpha+1}H^{\alpha+2}}{\alpha+2}\left(1-\frac{1}{\mathrm{Ca}\,T^2}\frac{d^3H}{dz^3}\right)^{\alpha}=1-H-\frac{T^2}{3}(1-H^3)-\frac{T_1^{\alpha+1}}{\alpha+2}, \qquad (4.2)$$

for the stagnation line

$$(\alpha-1)^{\alpha}(\alpha+1)\,T^{2\alpha}H_s^{2(\alpha-1)}\left[H_s-\frac{3}{\alpha-1}\left(\alpha+2-H_s-\frac{\alpha+2}{3}T^2-T_1^{\alpha+1}\right)\right]=$$
$$=6^{\alpha}T_1^{\alpha+1}\left(\alpha+2-H_s-\frac{\alpha+2}{3}T^2-T_1^{\alpha+1}\right)^{\alpha},$$

where

$$T=h_0\,(a\rho g/U)^{\frac{1}{2}};\; T_1=h_0\left[(\rho g)^{\alpha}\,b/U\right]^{\frac{1}{\alpha+1}};\; \mathrm{Ca}=\frac{U}{a\sigma};\; \mathrm{Ca}_1=\left(\frac{Uh_0^{\alpha-1}}{b\sigma^{\alpha}}\right)^{\frac{1}{\alpha}};\; \mathrm{Ca}\,T^2=\mathrm{Ca}_1 T_1^{\frac{\alpha+1}{\alpha}}.$$

In Fig. 3 we have plotted the results of calculations based on Eqs. (2.8), (3.2), and (4.2) and the experimental points of [1] for a 0.16% aqueous solution of Carbopol, whose rheological behavior is described [11] by the power law (n = 0.56, k = 0.6 $N\cdot sec^{n}\cdot m^{-2}$) and the Ellis model [α = 2.01, a = 0.129 $m^2\ (N\cdot sec)^{-1}$, b = 0.014 $m^{2\alpha}\cdot N^{-\alpha}\cdot sec^{-1}$]. The theoretical curves (I denotes the power law, II the Ellis model) for the film thickness coincide almost completely and lie considerably above the experimental data. This indicates the legitimacy of using the simpler power law instead of the three-parameter Ellis model in withdrawal problems and the need to seek another reason for the discrepancy between theory and experiment.

5. Experiments [12, 13] on the gravity flow of films of polymer solutions along an inclined plane showed that there is a wall effect. In the immediate vicinity of the wall the moving medium separates, forming a very thin wall layer with reduced polymer concentration as compared with the rest of the flow, which slides over this layer as if it were a lubricant. In the capture process the film is entrained by the moving surface and, at the same time, flows along it under the action of gravity. It is therefore natural to assume [14] that the entrainment of rheologically complex media involves a wall effect.

Taking this into account, we will construct a quantitative theory of liquid capture. We assume that the effective rate of slip u_c at the wall is uniquely determined by the local shear stress at the wall τ_w [12]. We assume a simple linear relation $u_c = \beta|\tau_w|$, where β (the slip coefficient) depends on the type of polymer and its concentration.

Then the dynamic meniscus of a power-law fluid is described by Eqs. (1.1)-(1.5), except for the first of conditions (1.2), which now takes the form $u = U - u_c$ at $y = 0$. Carrying out the calculations in the same way as above, for the dynamic meniscus we obtain

$$u=U-u_c-\frac{n}{n+1}\left(\frac{\rho g}{k}-\frac{\sigma}{k}\frac{d^3h}{dx^3}\right)^{1/n}\left[h^{\frac{n+1}{n}}-(h-y)^{\frac{n+1}{n}}\right],$$

$$\tau_w = -\left(\rho g - \sigma \frac{d^3 h}{dx^3}\right) h = -k\left[\frac{2n+1}{nh}\left(1 - \frac{h_0}{h}\right)(U - u_c) + \left(\frac{\rho g}{k}\right)^{\frac{1}{n}} h_0^{\frac{2n+1}{n}} \Big/ h^2\right]^n,$$

$$Q = (U - u_c)\, h_0 - \frac{n}{2n+1}\left(\frac{\rho g}{k} h_0^{2n+1}\right)^{1/n} = (U - u_c)\, h - \frac{nk}{2n+1}^{2n+1}\left(\frac{\rho g}{k} - \frac{\sigma}{k}\frac{d^3 k}{dx^3}\right)^{1/n},$$

$$h_s = \frac{2n+1}{n} h_0 - \frac{h_0^{(2n+1)/n}}{U - u_c}\left(\frac{\rho g}{k}\right)^{1/n}.$$

After some simple manipulation, going over to the dimensionless quantities of (1.9), we find the equations for the film thickness, slip rate, and stagnation line, respectively:

$$\frac{H^{2n+1}}{\mathrm{Ca}}\frac{d^3 H}{dz^3} = H^{2n+1} T^{n+1} - \left[\frac{2n+1}{n}(1 - v_c)(H - 1) + T^{\frac{n+1}{n}}\right]^n; \tag{5.1}$$

$$v_c H^{2n} = F\left[\frac{2n+1}{n}(1 - v_c)(H - 1) + T^{\frac{n+1}{n}}\right]^n, \quad H_s = \frac{2n+1}{n} - T^{\frac{n+1}{n}} \Big/ (1 - v_c|_{H_s}). \tag{5.2}$$

Here $v_c = u_c/U$, $F = \beta k U^{n-1} h_0^{-n}$ are the dimensionless effective slip rate and slip coefficient.

Taking into account the wall effect, as distinct from the dynamic meniscus, does not affect either the shape of the surface (2.8) in the static meniscus zone or the joining condition (3.1).

The solution algorithm remains as before. We assign the parameters Ca and F and select a starting value of T. For the boundary conditions (1.11), using relation (5.2), we integrate Eq. (5.1) and check the satisfaction of joining condition (3.2) by means of the solution (2.8). By iteration we find the value of T for which condition (3.2) is satisfied. The results of the calculations are presented in Fig. 2 [curve V for n = 0.6, F = 1; curve VI for n = 0.4, F = 2) and Fig. 4 (the continuous curves are for n = 1, the broken curves for n = 0.5; 1) F = 0, 2) F = 1, 3) F = 2]. As may be seen from Fig. 2, the experimental points for pseudoplastic fluids are well described by the theoretical curves obtained taking into account the wall slip effect. Figure 4 shows that an increase in the slip coefficient reduces the thickness of the entrained film. At intermediate withdrawal rates $10^{-2} < \mathrm{Ca} < 10$ the Newtonian (n = 1) and all the pseudoplastic (n < 1) fluids have similar dependences in the dimensionless complexes Ca, T, and F. From Eqs. (5.2) for n = 1 it follows that the effective slip rate $v_c = F[3(H - 1) + T^2][3F(H - 1) + H^2]^{-1}$ varies from a minimum $v_c = FT^2$ in the zone of constant film thickness (H = 1) to a maximum $H_s = (3 - T^2)/2 + [(3 - T^2)^2/4 - 2FT^2]^{1/2}$ on the stagnation line.

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PRESSURE FLOW OF LIQUID WHICH CONGEALS ON A PIPE SURFACE UNDER CONDITIONS OF DISSIPATIVE HEAT RELEASE

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UDC 532.78+532.542

There are many known processes in nature and in engineering where the flow of liquid is accompanied by a phase transformation. Examples of such processes are the accidental overcooling of pipelines [1], the transport of highly paraffinous petroleum [2], the motion of magma along a dike [3], the high-velocity flow of gas past an object [4], or even the electrical heating of a conductor during phase transformation [5]. The substantial effect of volumetric heat release during phase transition is shown in [4, 5]. An important peculiarity in these examples is the simultaneous interaction of the phase transition with chemical, Joule, or dissipative heat release. Earlier consideration has been made of the effect of the phase transition on the critical conditions of thermal shock in planar [6] and in cylindrical [7] regions and of its hydrodynamic analog, Couette flow [8].

This study investigates the peculiarities of the phase transition under conditions of viscous liquid pressure flow inside a pipe of circular cross section and of infinite length where there exists a given pressure gradient and a given flow rate. Either a constant temperature or a constant thermal flow is applied to the wall of the pipe.

It was shown that in all ranges of the parameters for the problem, a steady-state solution is achieved. Steady-state temperature and velocity profiles are determined. For a given pressure gradient and wall temperature in the quasisteady-state approximation, a plot is given for ranges of the parameters corresponding to the characteristic type of flow: a steady-state condition with intermediate positioning of the phase boundary, the condition of pipe capping (total phase transformation), and the condition of hydrodynamic thermal shock [9]. It was shown that for a given thermal flow on the wall the condition for intermediate positioning of the phase boundary is absent.

The peculiarities of flow for a given flow rate are analyzed. In this case, the steady-state flow with intermediate positioning of the phase boundary always exists. For a given thermal flow on the wall of the pipe it is possible to have flow without the solid phase. The flow rate and pressure characteristics are obtained, and the effect of phase transition is discussed.

1. Statement of the Problem. We will consider a phase transition of the first kind under the conditions of viscous, Newtonian liquid pressure flow inside a pipe of circular cross section and of infinite length whose walls are maintained at a constant temperature T_0 which is less than the temperature of phase T_*. Because of cooling, the liquid solidifies and a phase division is created on the inner surface at $r = r_*$. The dependence of viscosity on temperature goes according to the law of Arrhenius: $\eta = \eta_0 \exp(E/RT)$, where η_0 is a pre-exponential factor, E is the initiation energy of viscous flow, R is the universal gas constant, and T is temperature.

The equations of motion and heat balance, taking into account dissipative heat release, and the boundary conditions, can be written in the form

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